Brook House Junior School
Progression in Written Methods for Addition

These notes show the stages in building up to a compact, efficient method for addition. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

Children need to be able to:

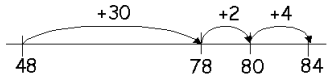
- recall addition pairs to 9 + 9
- know all complements to 10
- add mentally a series of single-digit numbers, such as 5 + 8 + 4
- count on in 1s, 10s and 100s
- partition numbers in ways other than into tens and ones to help with bridging multiples of 10 and 100

Empty Number Line

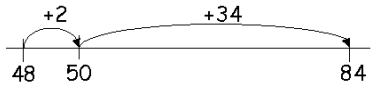
The empty number line helps to record the steps on the way to calculating the total.
The steps often bridge through a multiple of 10.

Example:

48 + 36 = 84



or:



Children need to be able to:

- partition numbers into hundreds, tens and ones
- recall addition pairs to 9 + 9
- add multiples of 10 or 100 (such as 60 + 70 or 600 + 700) using a related fact (6 + 7) and knowledge of place value
- mentally add multiples of 100, 10 and 1 e.g. 800 + 130 + 12

Partitioning

When adding larger numbers, it becomes less efficient to count on so partitioning is used.
Partition into (hundreds) tens and ones, add to form partial sums and then recombine.

47 + 76 = 40 + 7 + 70 +6 40 + 70 = 110 7 + 6 = 13 110 + 13 = 123	76 + 47 76 + 40 = 116 116 + 7 = 123

Partitioning all the numbers mirrors the standard column method where ones are placed under ones and tens under tens etc.

Example:

Partitioned numbers are written under one another:

47 + 76 = 40 + 7
= 70 + 6
110 + 13 = 123

375 + 567 = 300 + 70 + 5
500 + 60 + 7
800 + 130 + 12 = 942

Expanded Column Method

The expanded method leads children to the more compact column method so that they understand the structure and efficiency of it.

The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Example:
Write the numbers in columns:

Add the ones first
TU 47 + 76 13 (6 + 7) 110 (40 + 70) 123

Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to consistently adding the ones digits first.
The addition of the tens in the calculation 47 + 76 is described as 'Forty plus seventy equals one hundred and ten', stressing the link to the related fact 'Four plus seven equals eleven'.

Column Method

The method is then shortened and when the column total is a two-digit number, the tens (or hundreds) are carried over into the next column. Use the words 'carry ten' or 'carry one hundred', **not** 'carry one'.

Example:

HTU 366 + 458 824 1 1

Once learned, this method is quick and reliable. Later, extend to adding three two-digit numbers, two three-digit numbers, and numbers with different numbers of digits. This method of can also be used to add decimals.

Brook House Junior School
Progression in Written Methods for Subtraction

These notes show the stages in building up to a compact, efficient written method for subtraction. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

Children need to be able to:
- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact (16 – 7) and their knowledge of place value
- know all complements to 10 and 100

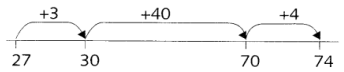
Children need to be able to:
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one
- partition numbers in different ways. e.g. 74 into 70 + 4 or 60 + 14
- subtract mentally a single-digit number or a multiple of 10 from a two-digit number
- add the totals (of the hundreds, tens and ones columns) mentally

Empty Number Line

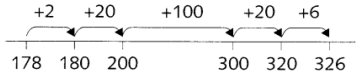
Empty or numbered lines are a useful way of modelling processes such as bridging through multiples of ten. The steps can be recorded by counting on or back.

Counting on example:

74 – 27 = 47



326 – 178 = 148



Counting back example:

15 – 7 = 8

74 – 27 = 47

The steps may be recorded in a different order or combined. With practice children will record less information and decide whether to count on or back.

Expanded column method

2 digit numbers with1 adjustment needed: 74 – 27 =

Partition into tens and ones. Then say, "There is not enough to subtract 7 from 4", rather than, "You ***can't*** subtract 7 from 4".

T	U
70	+ 4
– 20	+ 7

70 + 4 is then readjusted to become 60 + 14. The calculation can now be carried out.

T	U
60	+ 14
– 20	+ 7
40	+ 7

3 digit numbers with1 adjustment needed: 563 – 271 =

Partition into hundreds, tens and ones.

500	+ 60	+ 3
– 200	+ 70	+ 1

500 + 60 needs to be adjusted to become 400 + 160. The calculation can now be carried out.

400	+ 160	+ 3
– 200	+ 70	+ 1
200	+ 90	+ 2

3 digit numbers with 2 adjustments needed: 563 – 278 =

This occurs when the tens ***and*** the ones to be subtracted are larger than those you are subtracting from

500	+ 60	+ 3
– 200	+ 70	+ 8

Firstly, 60+3 is adjusted to become 50+13

500	+ 50	+ 13
– 200	+ 70	+ 8

500+50 is then adjusted to become 400+150. The calculation can now be carried out.

400	+ 150	+ 13
– 200	+ 70	+ 8
200	+ 80	+ 5

3 digit numbers with zeros where 2 adjustments are needed: 503 – 278 =

When 0's are involved, the adjustments need to be done in a different order. There is not enough to subtract 8 from 3

500	+ 0	+ 3
– 200	+ 70	+ 8

As there are no tens, 500 + 0 is adjusted first to become 400 + 100

400	+ 100	+ 3
– 200	+ 70	+ 8

Then 100 + 3 can be adjusted to 90 + 13. The calculation can now be carried out.

400	+ 90	+ 13
– 200	+ 70	+ 8
200	+ 20	+ 5

Over time, recording is refined

2 digit numbers with1 adjustment needed: 74 – 27 =

60 14
~~70~~ + ~~4~~
– 20 + 7
40 + 7

3 digit numbers with1 adjustment needed: 563 – 271 =

400 160
~~500~~ + ~~60~~ + 3
– 200 + 70 + 1
200 + 90 + 2

3 digit numbers with 2 adjustments needed: 563 – 278 =

150
400 ~~50~~ 13
~~500~~ + ~~60~~ + ~~3~~
– 200 + 70 + 8
200 + 80 + 5

3 digit numbers with zeros where 2 adjustments are needed: 503 – 278 =

90 13
400 ~~100~~
~~500~~ + ~~0~~ + ~~3~~
– 200 + 70 + 8
200 + 20 + 5

Over time, recording is refined

2 digit numbers with1 adjustment needed: 74 – 27 =

$^{6}\cancel{7}^{1}4$
$-\ 2\ 7$
$4\ 7$

Say, "60 – 20" or, "6 tens – 2 tens" not, "6 – 4"

3 digit numbers with1 adjustment needed: 563 – 271 =

$^{4}\cancel{5}^{1}6\ 3$
$-\ 2\ 7\ 1$
$2\ 9\ 2$

Say, "60 – 20" or, "6 tens – 2 tens" not, "6 – 4"

3 digit numbers with 2 adjustments needed: 563 – 278 =

$^{4}5^{15}\cancel{6}^{\ 1}3$
$-\ 2\ 7\ 8$
$2\ 8\ 5$

Say, "150 – 70" or "15 tens – 7 tens" not, "15 – 7"

3 digit numbers with zeros where 2 adjustments are needed: 503 – 278 =

$^{4}\cancel{5}^{9}\cancel{0}^{1}3$
$-\ 2\ 7\ 8$
$2\ 2\ 5$

Say, "100 – 70" or, "10 tens – 7 tens" not, "10 – 7"

Brook House Junior School
Progression in Written Methods for Multiplication

These notes show the stages in building up to a compact, efficient method for multiplication. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

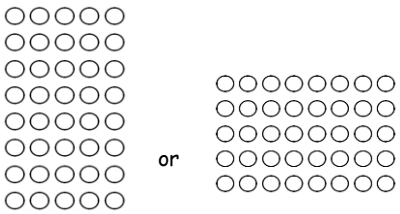
Children need to be able to:
- count in steps
- understand multiplication as repeated addition

Children need to be able to:
- partition numbers into multiples of one hundred, ten and one and in other ways
- recall multiplication facts to 10 × 10
- work out products such as 70 × 5, 70 × 50, 700 × 5, or 700 × 50, using the related fact, 7 × 5, and an understanding of place value
- add combinations of numbers mentally or using a written method

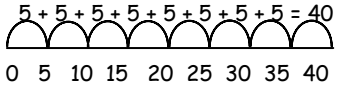
Method 1 – Repeated addition

Children start by understanding multiplication as arrays and repeated addition. They use this understanding to help them work out multiplication facts they cannot recall quickly

Example:
For '8 x 5', children picture:

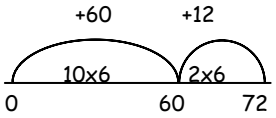


They use repeated addition to work out the calculation:



Recording of the steps on the number line may be refined as understanding and knowledge of facts develops:

Example:
12 x 6



This will support children in learning their tables using known facts and in understanding the distributive law which they will apply later when using the grid method.

Method 2 – Grid method into short/long multiplication

Multiplications can be carried out using the law of distribution which allows the numbers to be partitioned and each part to be multiplied separately. The products are then added to find the total product.

Stage 1 – Grid method

When multiplying a 1-digit number by a 2-digit number, children partition the numbers

38 X 7

X	7
30	210
8	56
	266

Ensure that children understand the relationship between 7x3 and 7x30 and are not simply 'adding a nought'

The same method can also be applied when multiplying a 1-digit number by a 3-digit number:

549 X 6

×	6
500	3000
40	240
9	54
	3294

Ensure that children understand the relationship between 6x5 and 6x500 and are not simply 'adding 2 noughts'

When multiplying a 2-digit number by a 2-digit number:

(1) Partition both numbers and multiply each part

X	20	7
50	1000	350
6	120	42

(2) Add the answers in each row

X	20	7	
50	1000	350	1350
6	120	42	162

(3)Add the two row totals to find the final product

X	20	7	
50	1000	350	1350
6	120	42	162
			1512

Stage 2 – Short/long multiplication (only for most able)

```
  38
× 7
  56
 210
 266
```

Children describe what they are doing by referring to the value of the digits. Say, "30x7", not "3x7" although the relationship should be stressed

56 X 27

```
  5 6
X 2 7
    4
-------
3 9 2
1 1 2 0
  1
-------
1 5 1 2
  1
```

Brook House Junior School
Progression in Written Methods for Division

These notes show the stages in building up to a compact, efficient method for division. Our aim is that children use mental methods when appropriate but for calculations that they cannot do in their heads they choose an appropriate written method which they can use accurately and with confidence. Time must be taken building up to the most efficient method to ensure complete understanding at each stage.

Children need to be able to: ▪ understand division as grouping and sharing ▪ understand multiplication and division as inverse operations ▪ recall multiplication and division facts to 10 × 10 ▪ understand remainders ▪ derive larger multiples using known facts e.g. 10 x 3 = 30 ➔20 x 3 = 60 etc ▪ add multiples mentally and work out differences	Use with the most able children who have a secure understanding of all the previous steps.	Use with the most able children who have a secure understanding of all the previous steps.
Stage 1 - Repeated addition	**Stage 2 - Short Division**	**Stage 3 – Long Division**
When it is not appropriate to use a sharing method for division and the division fact is not known, repeated addition (using the relationship between multiplication and division) can be used. **Example without remainder:** 40 ÷ 5 fives Ask "How many 5s in 40?" 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 8 0 5 10 15 20 25 30 35 40 **Example with remainder:** 38 ÷ 6 with a remainder of 2 6 + 6 + 6 + 6 + 6 + 6 + 2 = 6 sixes 0 6 12 18 24 30 36 38 **Mental Method Only** For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts. **Example without remainder:** 81 ÷ 3 30 30 21 10 10 7 0 30 60 81 This could either be done by working out the numbers of threes in each jump as you go along (10 threes are 30, another 10 threes makes 60, and another 7 threes makes 81. That's 27 threes altogether) or by counting in jumps of known multiples of 3 to reach 81 (30 + 30 + 21) then working out the number of threes in each jump. **Example with remainder:** 158 ÷ 7 70 70 14 2 10 10 2 0 70 140 154 158 10 sevens are 70, add another 10 sevens is 140, add 2 more sevens is 154 add 2 makes 158. So there are 22 sevens with a remainder of 2. The remainder is indicated above the jump rather than inside it, so that children do not mistakenly add 10, 10, 2 and 2 and get an answer of 24.	**Example without remainder:** 81 ÷ 3 2 7 3 ⟌ 8²1 Children use their knowledge of the 3 times table to find, "How many 3s in 80 where the answer is a multiple of 10?" This gives 20 threes (since 30 threes would be too many), with 20 remaining (2 tens are carried over to the next column) Now ask: 'How many threes in 21". **Example with remainder:** 4 7 r 2 6 ⟌ 2 8⁴4 Once children's understanding of this method is secure they might shorten their dialogue to: "How many 6s in 28?" "4 remainder 4" "How many 6s in 44?" "7 remainder 2" **BUT** ensure children have a secure understanding of what they are doing and are able to use their knowledge of related facts to either make a rough estimate first or have an idea about whether their final answer is reasonable or not.	25 r 16 36 ⟌ 916 - 720 (36X20 = 720) 196 - 180 (36X5 = 180) 16